# Lec 6: Friday, 23rd August - Linear SVMs continued

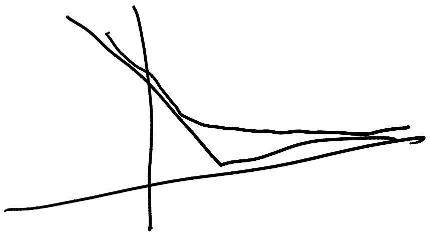
20 August 2013
12:58

$$\frac{1}{2} w^t w + C \sum_{i=1}^{N} \log \left(1 + e^{1 - y_i w^t x_i}\right)$$

$\equiv$

$$\min_{w,\xi} \frac{1}{2} w^t w + C \sum_{i=1}^{N} \log \left(1 + e^{\xi_i}\right)$$

$$\xi_i \geq 1 - y_i w^t x_i$$



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PEGASOS

1)

$$f(w) = \frac{\lambda}{2} w^t w + \frac{1}{m} \sum_{(x,y) \in S} l(w, x, y)$$

$T, K$

$A_t \subseteq S \quad |A_t| = k$

$$f(w_t; A_t) = \frac{\lambda}{2} w_t^t w_t + \frac{1}{|A_t|} \sum_{x,y \in A_t} l(w, x, y)$$

$$\nabla_t = \lambda w_t - \frac{1}{|A_t|} \sum_{x,y \in A_t} x y_{-1}$$

$$w_{t+1/2} = w_t - \eta_t \nabla_t$$

$$w_{t+1} = \min\left\{1, \frac{1}{\sqrt{\lambda}\, \|w_{t+1/2}\|}\right\} w_{t+1/2}$$

$$w^* = \arg\min f(w)$$

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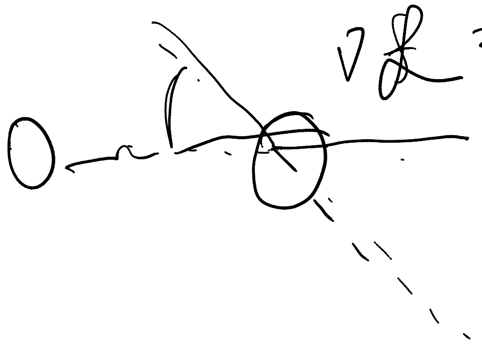
$$f(w_1) - f(w^*) \leq \epsilon$$

in $\tilde{O}\left(\frac{1}{\lambda \epsilon}\right)$ iterations

$\nabla \mathcal{L} = 0$



Lemma 1

$$\frac{1}{T} \sum_{t=1}^{T} f_t(w_t) \leq \frac{1}{T} \sum_{t=1}^{T} f_t(u) + \frac{G^2 (1 + \ln(T))}{2 \lambda T}$$

1) $f_t$ is $\lambda$-strongly convex

2) $\|\nabla_t\| \leq G$

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3) $u \in B$

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$f(w_t; (x_i, y_i)) = \frac{\lambda}{2} \|w\|^2 + \frac{1}{k} \sum \ell(w_t; x_i, y_i)$

$\|\nabla_t\| \leq G$

$\|\nabla_t\| \leq (\sqrt{\lambda} + R)$

$\nabla_t = \lambda \underline{w} - \frac{1}{k} \sum \mathbb{1}(1 - y_i w^t x_i > 0) \, y_i \underline{x_i}$

$\|a + b\| \leq \|a\| + \|b\|$

$\|w\| \leq \frac{1}{\sqrt{\lambda}}$

$\|\lambda w\| \leq \sqrt{\lambda}$

$\|x_i\| \leq R$

$\left\| \frac{1}{k} \sum \mathbb{1}(1 - y_i w^t x_i > 0) y_i x_i \right\| \leq \underbrace{\|x_i\|}_{\leq R}$

$$\| \nabla_t \| \leq \left( \sqrt{\lambda} + R \right)$$

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Assumption 3

$$\| w^* \| \leq \frac{1}{\sqrt{\lambda}}$$

$$\frac{1}{T} \sum_t f(w_t) \leq \frac{1}{T} \sum_{t=1}^{T} f(w^*) + \frac{c\,(1 + \ln(T))}{2 \lambda T}$$

$$f\left( \frac{1}{T} \sum_t w_t \right) \leq \frac{1}{T} \sum_t f(w_t)$$